Flow rate measurement in modern ambient air samplers – how accurate?

Presented by: Mark A. Weitz, P.E.
Chinook Engineering
555 Absaraka
Sheridan, Wyoming 82801 USA
(307) 672-7790
chinook@warmwind.com

Shaun S. Wulff, Ph.D.
Associate Professor of Statistics
University of Wyoming
Laramie, WY 82071 USA
(307) 766-6483
wulff@uwyo.edu

ABSTRACT

New generation ambient air samplers operate at much lower sampling flow rates than do their "high volume" predecessors, namely 16.7 liters/minute. When these new air samplers emerged several years ago there were limited tools available with which to assess and calibrate their operating flow rate. In 1996 a flow meter specifically designed for these new air samplers came available, known as the Streamline™ Flow Transfer Standard. With the establishment of a nation wide PM2.5 sampling network in 1999 which use exclusively 16.7 liter/minute samplers, Streamlines have come into wide usage. This paper addresses the "accuracy" of flow measurements produced by Streamline FTSs, and the subsequent accuracy of the flow rates of air samplers calibrated with Streamlines.

A statistical uncertainty analysis was performed for the Streamline FTS used at a nominal flow rate of 16.5 liters/minute, yielding an uncertainty in its measured flow rate of ±0.67% which corresponds to ±0.11 liters/minute. A subsequent uncertainty analysis was performed for the flow rate of an ambient air sampler using a Streamline and a typical field instruments, yielding an uncertainty of ±0.98% (±0.16 liters/minute) at 16.7 liters/minute.

I. INTRODUCTION

Sample volume is an essential parameter in the determination of ambient air pollutant concentrations. Most modern air samplers compute total sample volumes by integrating a continuous volumetric flow rate measurement over the duration of a sample period. Continuous flow measurements are accomplished with on-board devices such as mass flow meters or dry gas meters, accompanied by ambient temperature and pressure measurements to account for ambient air density. By good scientific practice, and by U.S. EPA requirements, these flow rate parameters measured and recorded by the air samplers must be checked periodically against independent devices known as "transfer standards".

The Streamline Flow Transfer Standard serves as the flow transfer standard in many air monitoring networks which use newer 16.7 liters/minute samplers such as PM2.5 Federal Reference Method manual samplers and TEOMs™. It is the purpose of this paper to provide air quality professionals a method to approximate the uncertainty in their sampler flow rate measurements.

Metrology Concepts

Instruments which are used to calibrate other instruments are commonly referred to as "transfer standards". Transfer standards are typically compared or "calibrated" against "primary standards" which are maintained and operated under controlled conditions such as a laboratory. It is desirable for a primary standard to be directly comparable or "traceable" to recognized "identity standard" which has a known and accepted value, and is considered the ultimate source for all descendent measurements.

In the U.S., the National Institute of Standards and Technology (NIST) maintains recognized identity standards. Transfer standards, calibrated against primary standards, calibrated against identity standards suggests an hierarchy of measurement traceability. Each calibration in the hierarchy introduces error. It is thus desirable to minimize the number of calibration steps between an identity standard and descendent transfer standards.

Some measurements, such as the height of a liquid as a pressure measurement are based on fundamental physical principles, are themselves considered "primary" and require no identity standard. Other measurements, like mass or temperature, depend on an identity standard with a mutually accepted value against which other masses can be compared. Still other measurements, like flow rate, are derived from a combination of measurements and assumptions, and as such have no discrete identity standard.

Measurement Uncertainty

All measurements have errors. The errors may be positive or negative, of variable magnitude, and may vary with time. Those which can be observed to vary during a test are called random errors. Those which remain constant, or apparently constant during a test are called biases, or systematic errors. Actual errors are rarely known; however uncertainty intervals can be estimated as upper bounds on the errors. The problem is to construct an uncertainty interval which models these errors[6]. Often, measuring instruments are adjusted or calibrated against reference standards to eliminate systematic effects. However, the uncertainties associated with these reference standards must also be accounted for[1].

Many methods are employed for addressing measurement uncertainties. It is desirable to utilize common, comparable methodologies, and to that end the International Organization for Standardization (ISO) and the American National Standards Institute (ANSI) have developed Standards[1,6] for estimating measurement uncertainties, both generally and for fluid flow specifically. However the subject is complex, involving both engineering and statistics. The 1984 ANSI Standard states: "…it must be recognized that no single method will give a rigorous, scientifically correct answer for all situations. Further, even for a single set of data, the task of finding and proving one method to be correct is almost impossible"[6]. Quoting further from the 1997 ISO Standard "error is an idealized concept and errors can not be known exactly"[1].

Flow Metrology

In contrast to measurements such as pressure, mass, time, there are no identity standards for flow rate - no "golden liter per minute" against which to compare all flow meters. Flow rate is a "derived" standard, based on a combination of mass, temperature, pressure, time and/or dimensional measurements, along with empirical knowledge of fluid properties[4,5]. A flow rate “primary standard” is an apparatus which includes a fluid mover, fluid reservoir, leak-free conduits for directing flow, a flow measurement element and accompanying instrumentation for determining fluid properties (e.g. pressure, temperature, viscosity, etc.). Each component of the apparatus, the operator, assumptions about the physical components and fluid properties, and the calculations which produce final calibration relationships contribute uncertainties in a flow rate primary standard apparatus, and to its descendent transfer standards.

Streamline FTSs are calibrated against a primary standard apparatus which employs critical flow venturis (CFVs) as the flow rate element. CFVs, also known as “sonic nozzles are used by Chinook Engineering because they provide very stable flow conditions, have no moving parts, and are well characterized and recognized as accurate flow meters for air[3,5]. Chinook’s CFV-based calibration apparatus is described below, and depicted in Figure 2.

This paper employs a combination of Standard methodologies to estimate the uncertainty associated with the calibration of Streamlines and their application on ambient air samplers. This paper is not intended to be an exhaustive treatise on flow metrology, it does not attempt to trace every measurement component to its ultimate NIST identity standard. Rather, it is intended to be an understandable working model for use by air pollution professionals. While standard methodologies are similar, there are numerous differences and subtleties. When estimating many of the components of error, judgement and estimation must be employed. Those interested in more exhaustive treatments on measurement uncertainty are encouraged to explore the wealth of information developing on this subject, the ANSI[6] and ISO[1] Standards being excellent starting points.

II. THE STREAMLINE FTS and its USE

Streamline Flow Transfer Standards are mounted on the inlet of modern air samplers, in series with the on-board volumetric flow meters as depicted in Figure 1.

Streamline FTS mounted on an air sampler, connected to a manometer.

Figure 1

The Streamline is an orifice type flow meter with a patented construction which provides steady flow conditions, and prevents dimensional changes with temperature - problems which can significantly affect the performance of other flow meters, especially when used in the field. All Streamlines are calibrated through their intended range against the sonic nozzle primary calibration apparatus described below and illustrated in Figure 2. Using Bernoulli's model a unique calibration relationship is created for each Streamline. allowing use at all ambient conditions.

An example calibration relationship is shown in Equation 1. Equation 1 is a linear form of y = m(x) + b where "m" and "b" are the unique calibration constants.

$$Qa = 0.4022\sqrt{\frac{(\Delta P)(T_{amb})}{P_{amb}}} - 0.3255 \qquad \text{Equation 1}$$

where: Q_a = actual flow rate in liters/min, 0.4022 and –0.3255 are unique calibration constants ("m" and "b" respectively), ΔP = pressure drop across the Streamline in "H2O, T_{amb} = ambient temperature in Kelvins, P_{amb} = ambient barometric pressure in atmospheres. Equation 1 is a derivation of Bernoulli's equation shown in Equation 2.

$$Qa = (A)(Y)(Cd)\sqrt{\frac{\Delta P}{\rho}} \qquad \text{Equation 2}$$

where: Q_a = actual flow rate, A is orifice cross sectional area, Y is the gas expansion factor (thermodynamic term), Cd is the orifice discharge coefficient (frictional term), ΔP is the pressure drop across an orifice and ρ is fluid density (described in Equation 1 by the temperature and pressure of the ambient air).

Comparing Equations 1 and 2, the slope "m" (0.4022) in Equation 1 assumes the quantity [(A)(Y)(Cd)] is constant through a Streamline's calibrated range. This assumption is substantiated by the excellent linearity of Streamline calibrations through their calibrated range where r^2 is typically >0.9999, as calibrated in the laboratory. The orifice area (A) will not change with ambient fluid conditions due to the Streamline's patented construction. However, the expansion factor and discharge coefficient of air do certainly change as Streamlines are used in the field at widely varying temperatures, pressures and humidities. Uncertainties associated with Y and Cd are therefore included later in this paper where an example uncertainty analysis for an ambient air sampler is presented.

III. CHINOOK's PRIMARY FLOW METROLOGY APPARATUS

Chinook Engineering employs critical flow venturis (CFVs) as primary mass flow elements in its calibration apparatus, with associated high precision temperature and pressure instrumentation. Chinook's apparatus, illustrated in Figure 2, was designed following consultation with NIST flow metrologists. CFVs are recognized as being well characterized for accuracy, and due to the presence of a sonic cone, provide inherently stable flow[5,8,10,11]. Chinook's flow metrology apparatus is maintained in statistical quality control by use of internal standards.

Figure 2 illustrates that five measurements are collected for each calibration point; namely, the pressure and temperature of the fluid at the inlet of the sonic nozzle, the ambient temperature and pressure, and the pressure drop across the Streamline.

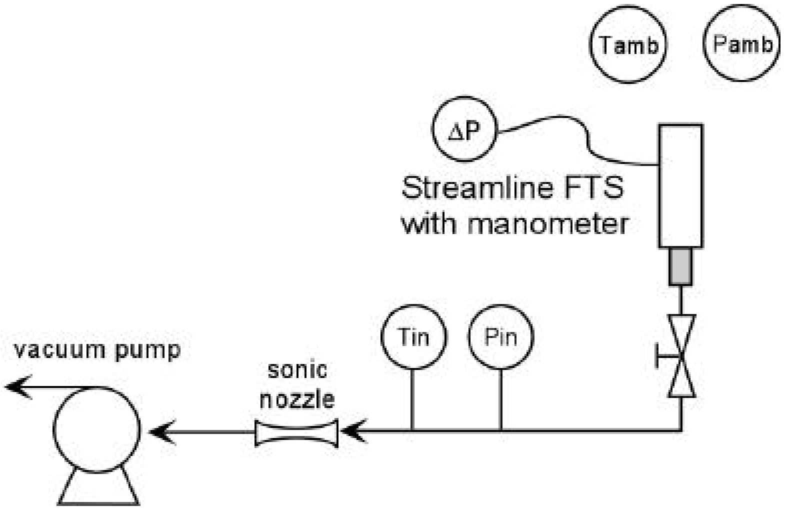

Figure 2. Chinook Engineering's sonic nozzle calibration apparatus (no scale).

IV. UNCERTAINTY ANALYSIS of a STREAMLINE FTS

As discussed in the Introduction there is no single best method for estimating the uncertainties associated with flow rate measurements. This paper uses a combination of techniques based primarily on ISO and ANSI Standards[1,6] modified for this application. A complete derivation of all equations and assumptions are not included here, the reader is referred to the references for comprehensive treatments.

The following uncertainty analysis entails:

1. Identification of elemental sources of uncertainty.
2. Estimation of standard uncertainties for each elemental source of uncertainty.
3. Combination of elemental components into standard uncertainties for each measurement component.
4. Calculation of sensitivity coefficients for each component of uncertainty from Taylor's series derivations of the defining measurement equations.
5. Calculation of an overall uncertainty including the uncertainty introduced by least squares regression using a linear structural model[7].

The defining equations which relate the flow rate of the CFV-based calibration apparatus to the pressure drop across the Streamline are as follows:

The mass flow rate of air through the CFV is given by[3]:

mass flow rate $$\dot{m} = \frac{(Pin)(A)(C^*)(Cd)}{\sqrt{(R)(Tin)}}$$ Equation 3 where: $\dot{m}$ is the mass flow rate, Pin and Tin are the pressure and temperature at the inlet of the CFV, respectively, A is the cross sectional area of the CFV, C* is a thermodynamic state variable that is a function of the fluid pressure and temperature in the nozzle throat, Cd is the discharge coefficient of the nozzle and R is the gas constant with appropriate units.

The mass flow rate through the CFV can be converted to a volumetric flow rate through the Streamline as follows:

volumetric flow rate
$$\mathrm{Qa} = \frac{\dot{m}}{\rho} \qquad \text{Equation 4}$$

where Qa is volumetric flow rate and ρ is the density of the ambient air.

The volumetric flow rate through a Streamline is fit to a linear form of Bernoulli's model as follows:

$$\mathrm{Qa} = m\left[\sqrt{\frac{(\Delta P)(T_{amb})}{P_{amb}}}\right] + b \qquad \text{Equation 5}$$

where: ΔP is the pressure drop across the Streamline at the discrete flow rates Qa measured with the CFV apparatus. Tamb and Pamb are the ambient temperature and pressure, respectively.

Equation 5 is in the form of a standard linear equation y = m(x) + b where "y" represents the discrete flow rates Qa provided by the CFV apparatus, and "x" represents the associated quantity under the radical. "m" and "b" are the slope and intercept obtained by fitting the calibration points by linear-least-squares through the calibrated range. The measurement uncertainties associated with a Streamline calibration are thus embodied in the calibration constants "m" and "b".

Combining Equations 3 and 4 obtains the operating equations for Qa in units of the measured input parameters:

$$\mathrm{Qa} = \frac{\dot{m}}{\rho} = \frac{(\mathrm{Pin})(\mathrm{A})(\mathrm{C^*})(\mathrm{Cd})}{\sqrt{(\mathrm{R})(\mathrm{Tin})(}\rho)} = (\mathrm{K})\frac{(\mathrm{Pin})(\mathrm{Tamb})}{\left(\sqrt{\mathrm{Tin}}\right)(\mathrm{Pamb})} \qquad \text{Equation 6}$$

where K combines A, C* and Cd along with the ideal gas constant and appropriate unit conversions.

Elements of Uncertainty

The manufacturer of Chinook's CFVs calibrate the nozzles to NIST traceable mass and time identity standards, and provides uncertainty estimates which include all parameters in Equation 3 (see Figure 2). The manufacturer's stated uncertainty for the CFV's performance will thus be used for the combined uncertainty of A, C*, Cd and R as a defined parameter "K". Pin and Tin are measured independently at Chinook Engineering's facility hence these instruments are included here as elemental sources of error. The ambient pressure and temperature instruments, and the precision manometer used to measure pressure drops across Streamlines are also identified as sources of uncertainty.

Instruments have one or several components of uncertainty which include the repeatability of measurements, instrument resolution, calibration hierarchies, and others as detailed in the analysis. The products of these elemental uncertainties are combined into an instrument uncertainty by summing the square root of the sum of the squares of the elemental uncertainties, a standard technique for propagation of error analyses[2,11,12].

Once raw measurements are taken the calibration data are fit by linear least squares to Bernoulli's model. The process of fitting measurements which contain uncertainties by a least squares algorithm introduces additional uncertainty. The least squares fit is therefore identified as another component of uncertainty and is treated by the use of a linear structural model[7].

Referring to Equations 2 and 5, fitting Qa versus SQRT [(ΔP)(Tamb)/Pamb)] into a linear model presumes the discharge coefficient (Cd) and expansion factor (Y) are constant through the calibrated flow range, manifested in the calibration constants "m" and "b". The validity of this is assumption is substantiated by the excellent linearity of Streamline calibrations (r^2>0.9999). However, in reality these parameters do change with flow rate and fluid density[8].

Because of the constancy of ambient conditions in the lab it is assumed that the slight variations in Cd and Y which might occur during a calibration are negligible, and are therefore not included in the uncertainty analysis for a Streamline. However, end-users of Streamlines measure flow rates at a variety of ambient temperatures and pressures. An example uncertainty analysis for an ambient sampler presented later in this paper does therefore incorporate components of uncertainty for Cd and Y.

The Streamline uncertainty analysis is concerned with the following components:

1. CFV performance characteristics
2. Pin, measurement of nozzle inlet pressure
3. Tin, measurement of nozzle inlet temperature
4. ΔP, measurement of pressure drop across Streamline
5. Tamb, measurement of ambient temperature
6. Pamb, measurement of ambient pressure
7. linear least squares fit of Qa ("y") to SQRT [(ΔP)(Tamb)/(Pamb)] ("x")

Each component of uncertainty has a "sensitivity" or weight which it carries into the overall Streamline uncertainty depending on its function in the defining equations, and the relative magnitude of the raw measurements. Sensitivity coefficients are estimated for the individual components based on a Taylor's series expansion of their defining equations. A linear structural model[7] is used to propagate the component uncertainties through the least squares algorithm to arrive at an overall uncertainty.

Uncertainty Analysis

Notes:

1) The linear structural model requires inputs of nominal values and uncertainties in raw measurement units, so much of the component uncertainty information presented here are in units specific to Chinook's calibration facility. In order to facilitate the use and comparison of these techniques and results, corresponding relative uncertainties derived from nominal conditions are also presented, expressed in parts per million (ppm) = per cent*10,000. Relative uncertainties are more readily combined into end-user analyses for their ambient air samplers.
2) The ISO Standard[1] makes distinctions between techniques used to analyze indvidual uncertainties -Type A and Type B - where Type A uncertainties are obtained through repeated measurements under controlled conditions; Type B are obtained through repeated use and operator judgment. Regarding Type A/Type B analyses, from the ISO Standard[1]: "…a Type B evaluation of standard uncertainty calls for insight based on experience and general knowledge, and is a skill that can be learned with practice. It should be recognized that a Type B evaluation of standard uncertainty can be as reliable as a Type A evaluation, especially in a measurement situation where a Type A evaluation is based on a comparatively small number of statistically independent observations". Due to limited statistical information, type B evaluations are used often in this paper's uncertainty analyses.
3) Rounding errors in unit conversions and truncation by electronic calculating devices also contribute uncertainty, but are generally recognized as being very small[1,7] and are ignored in the following analyses.

CFV Performance Characteristics

The CFV manufacturer states a standard uncertainty of 5000 ppm (0.5%) at a 95% confidence interval. A coverage factor of 2 is used for reporting this uncertainty, implying a standard uncertainty of 5000/2=2500 ppm which corresponds to 0.04 liters/minute of a nominal 16.7 liter/minute flow rate. This uncertainty is articulated through the parameter "K" defined in section IV.1. The nominal value for K is 1.6457E-3 (liter)(atm)/[sqrt(K)(mmHg)(min)] with corresponding uncertainty at 2500 ppm of 4.1E-6. This is a type A evaluation provided by the manufacturer based on the calibration and analysis of numerous nozzles.

Inlet Pressure Measurement (Pin)

The uncertainty in nozzle inlet pressure measurement is assumed to be made up of uncertainties in three components: the variation of repeated measurements, instrument resolution and the calibration standard used to maintain the instrument[1].

Variation of Observed Measurements

Regular observations indicate the variation of measurements under constant pressure is less than the resolution of the reading. The standard uncertainty is therefore conservatively assumed to be equal to the instrument resolution of 0.1 mmHg, which corresponds to 200 ppm of a 500 nominal mmHg reading. This is a type B evaluation.

Instrument Resolution
Instrument resolution is included in Variation of Observed Measurements above.

Calibration Standard
The electronic pressure instrument is calibrated against a mercury-in-glass absolute barometer whose manufacturer states an uncertainty of 200 ppm. The manufacturer's claim is interpreted as the maximum bounds within which all calibration values are said to lie. The probability distribution is conservatively assumed to be rectangular (multiply by 0.29) with a resulting standard uncertainty of 0.29(200) = 58 ppm, which corresponds to 0.029 mmHg of a nominal 500 mmHg reading. This is a type B evaluation.

Inlet Temperature Measurement (Tin)
The uncertainty in nozzle inlet temperature measurement is assumed to be made up of uncertainties in four components: the variation of repeated measurements, instrument resolution, the calibration standard used to maintain the instrument and frictional effects[1,3].

Variation of Observed Measurements
Regular observations indicate the variation of measurements under constant temperature is less than the resolution of the reading. The standard uncertainty is therefore conservatively assumed to be equal to the instrument resolution of 0.1K, which corresponds to 340 ppm of a nominal 294 K reading. This is a type B evaluation.

Instrument Resolution
Instrument resolution is included in Variation of Observed Measurements above.

Calibration Standard
The electronic thermometer is calibrated against a mercury-in-glass thermometer, whose manufacturer states an uncertainty of 0.3 K , the performance of which is confirmed by periodic ice point calibrations. The manufacturer's claim is interpreted as the maximum bounds within which all calibration values are said to lie. The probability distribution is conservatively assumed to be rectangular with a standard uncertainty of 0.29(0.3) = 0.087K which corresponds to 296 ppm of a nominal 294 K reading. This is a type B evaluation.

Frictional Effects
The thermocouple probe is heated by friction from the air passing over it. The heat is dissipated by conduction and radiation of the surrounding surfaces. The worst case standard uncertainty for these effects are estimated to be 0.01K and 0.01K, respectively[2]. The standard uncertainty of these effects is the root-sum-square of 0.01K and 0.01K = 0.014 K which corresponds to 48 ppm of a nominal 294K temperature. This is a type B evaluation.

Pressure Drop Measurement Across Streamline (DP)
A precision liquid manometer is used to measure pressure drops across a Streamline. Uncertainty is associated with the scale which is used to read the liquid height, and the

variation of repeated measurements. The height of a column of liquid is considered a primary standard for pressure, so this manometer does not depend on a calibration standard.

Variation of Observed Measurements

The variation of observed measurements is due to the operator’s abilities to discern a reading against the scale. Regular observations indicate the variation of measurements at constant pressure is 0.0079 “H2O (0.2 mmH$_2$O) which is assumed to be the standard uncertainty which corresponds to 1607 ppm for a nominal pressure drop measurement of 4.9 “H$_2$O (125 mm H2O). This is a type B evaluation.

Scale Accuracy

The manufacturer states an uncertainty in measurement of 200ppm. The manufacturer’s claim is interpreted as the maximum bounds within which all values are said to lie. The probability distribution is conservatively assumed to be rectangular with a standard uncertainty of 0.29(200)=58 ppm which corresponds to 2.9E-4 “H$_2$O (0.0073 mmH2O) of a nominal reading 4.9 “H$_2$O(125 mmH2O). This is a type B evaluation.

Ambient Temperature Measurement (Tamb)

The uncertainty in ambient temperature measurement is assumed to be made up of uncertainties in the variation of repeated measurements and calibration uncertainty[1].

Variation of Observed Measurements

The variation of repeated measurements is due to the operator’s abilities to discern a reading against the scale. Regular observations indicate the variation of measurements under constant temperature is 0.1 K which is assumed to be the standard uncertainty which corresponds to 340 ppm for a nominal temperature of 294K. This is a type B evaluation.

Calibration Standard

The ambient temperature thermometer is calibrated against a mercury-in-glass thermometer, whose manufacturer states an uncertainty of 0.3°K , the performance of which is confirmed by periodic ice point calibrations. The manufacturer’s claim is interpreted as the maximum bounds within which all values are said to lie. The probability distribution is assumed to be rectangular with and a standard uncertainty of 0.29(0.3) = 0.087K, which corresponds to 296 ppm for a nominal 294K ambient temperature. This is a type B evaluation.

Ambient Pressure Measurement (Pamb)

The uncertainty in ambient pressure measurement is assumed to be made up of uncertainties in three components: the variation of repeated measurements, instrument resolution and the calibration standard[1].

Variation of Observed Measurements

Regular observations indicate the variation of measurements under constant pressure is less than the resolution of the reading. The variation is therefore conservatively assumed to be equal to the instrument resolution of 1.32E-4 atmospheres(0.1 mmHg), the standard uncertainty being 0.1 mmHg which corresponds to 153 ppm of a 657 mmHg=0.865 atmospheres nominal pressure reading. This is a type B evaluation.

Instrument Resolution
Instrument resolution is included in Variation of Observed Measurements above.

Calibration Standard
The pressure instrument is calibrated against a mercury-in-glass absolute barometer. The manufacturer states an uncertainty of 200 ppm which is interpreted as the maximum bounds within which all calibration values are said to lie. The probability distribution is conservatively assumed to be rectangular with a standard uncertainty of 0.29(200) = 58 ppm which corresponds to 5.01E-5 atmospheres (0.038 mmHg) of a 0.865 atmosphere (657 mmHg) nominal barometric pressure. This is a type B evaluation.

Sensitivity Coefficients
The sensitivity coefficients associated with the above component uncertainties are estimated from Taylor's series derivations of Equation 6 for Qa, and Equation 5 for the SQRT[(ΔP)(Tamb)/(Pamb)]. As discussed earlier these quantities correspond to "y" and "x" which are fit by least squared to Bernoulli's model to produce the unique calibration constants "m" and "b" of a Streamline.

The first five derivatives are associated with the quantities measured to determine Qa values from the CFV apparatus. The value for "K", a combination of nozzle dimensions, fluid properties and unit conversions as explained earlier is:

$$K = 1.6457E\text{-}3 \frac{(\mathrm{liter})(\mathrm{atm})}{(\sqrt{K})(\mathrm{mmHg})(\mathrm{min})}$$

For CFV performance:

$$C_K = \left(\frac{\partial Qa}{\partial K}\right) = \left(\frac{(Pin)(Tamb)}{(\sqrt{Tin})(Pamb)}\right) = \left(\frac{(500)(294K)}{\sqrt{(294K)}(.865atm)}\right) = 9911.23\frac{(mmHg)\sqrt{K})}{(atm)}$$

for nozzle inlet pressure:

$$C_{Pin} = \left(\frac{\partial Qa}{\partial Pin}\right) = \left(\frac{(K)(Tamb)}{(\sqrt{Tin})(Pamb)}\right) = \left(\frac{(1.65E-3)(294K)}{\sqrt{(294K)}(.865atm)}\right) = 3.29E-2\frac{(liters)}{(mmHg)(min)}$$

for ambient temperature:

$$C_{Tamb} = \left(\frac{\partial Qa}{\partial Tamb}\right) = \left(\frac{(K)(Pin)}{(\sqrt{Tin})(Pamb)}\right) = \left(\frac{(1.65E-3)(500)}{\sqrt{(294K)}(.865atm)}\right) = 5.58E-2\frac{(liters)}{(K)(min)}$$

for nozzle inlet temperature:

$$C_{Tin} = \left(\frac{\partial Qa}{\partial Tin}\right) = \left(-\frac{(K)(Pin)(Tamb)}{2\left(Tin^{1.5}\right)(Pamb)}\right) = \left(-\frac{(1.65E-3)(500)(294)}{2(294)^{1.5}(.865)}\right) = -2.79E-2\,\frac{(liters)}{(K)(min)}$$

for ambient pressure:

$$C_{Pamb} = \left(\frac{\partial Qa}{\partial Pamb}\right) = \left(-\frac{(K)(Pin)(Tamb)}{\left(\sqrt{Tin}\right)(Pamb)^2}\right) = \left(-\frac{(1.65E-3)(500)(294)}{\left(\sqrt{294}\right)(.865atm)^2}\right) = -19.01\,\frac{(liters)}{(atm)(min)}$$

The last three derivatives are associated with the quantities measured at the Streamline. The quantity “x” is defined as x = SQRT[(ΔP)(Tamb)/(Pamb)].

For pressure drop across a Streamline:

$$C\Delta P = \left(\frac{\partial x}{\partial \Delta P}\right) = \left(\frac{1}{2}\sqrt{\frac{(Tamb)}{(Pamb)(\Delta P)}}\right) = \left(\frac{1}{2}\sqrt{\frac{(294K)}{(.865atm)(4.9\text{"}H2O)}}\right) = 4.13\sqrt{\frac{(K)}{(atm)}}$$

for ambient temperature:

$$CTa = \left(\frac{\partial x}{\partial Ta}\right) = \left(\frac{1}{2}\sqrt{\frac{(\Delta P)}{(Pamb)(Tamb)}}\right) = \left(\frac{1}{2}\sqrt{\frac{(4.9\text{"}H2O)}{(.865atm)(294K)}}\right) = 0.07\sqrt{\frac{(\text{"}H2O)}{(atm)}}$$

for ambient pressure:

$$CPa = \left(\frac{\partial x}{\partial Pa}\right) = -\frac{\sqrt{(\Delta P)(Tamb)}}{Pamb^2} = \left(-\frac{\sqrt{(4.9\text{"}H2O)(294K)}}{(0.865atm)^2}\right) = -23.86\sqrt{(\text{"}H2O)(K)}$$

Combining Uncertainties
Elemental uncertainties are combined by quadrature sum (square root of the sum of the squares) into overall component uncertainties as summarized in Table 1.

Linear Structural Model
In order to find the overall uncertainty for Qa, it is necessary to find the uncertainty associated with 'm' and 'b'. These regression coefficients have uncertainty associated with their variability, bias, and correlation. The explanatory variable, “x” in the regression corresponds to the square root quantity in Equation 5 which is subject to uncertainty associated with measurement error. The explanatory variable is also a random variable due to variations in ambient pressure and temperature. The response variable, “y”, in the regression corresponds to the quantity in Equation 6 which is also subject to uncertainty associated with measurement error. The associated uncertainties of “x” and “y” are potentially correlated. The quantities m and b are linear least squares estimates in a regression model with errors in both random variables. Such models are termed <u>linear structural models</u> (Reilman et al., 1986). Note, it is recognized there is

correlation between measured values in “x” and “y”. However, the exact nature and magnitude of the correlations is not known. For this analysis conservative estimates for correlations are assumed.

Table 1: Summary of uncertainty components and sensitivity components

Source of Uncertainty	standard uncertainty *u*	sensitivity coefficient C	nominal measurement	normalized uncertainty [ppm]	normalized sensitivity coefficient
CFV performance characteristics (as defined by parameter "K")	**4.11E-6**	9911	1.6457E-3	**2500**	1
Inlet pressure (Pin)	**0.104 mmHg**	0.0329	500 mmHg	**208**	1
variance of repeated measurements/instrument resolution	0.1			200	
calibration standard	0.029			58	
Inlet temperature (Tin)	**0.133 K**	-0.0279	294 K	**453**	½
variance of repeated measurements/instrument resolution	0.1			340	
calibration standard	0.087			296	
frictional effects	0.014			48	
Pressure drop across Streamline (ΔP)	**0.0079 "H2O**	4.13	4.9 " H2O	**1607**	½
variation of repeated measurements	0.0079				
Scale accuracy	2.9E-4				
Ambient Temperature (Tamb)	**0.133 K**	5.6E-2 for 'y' 0.07 for 'x'	294 K	**451**	1 for 'y' ½for 'x'
variation of repeated measurements	0.1			340	
calibration standard	0.087			296	
Ambient Pressure (Pamb)	**0.00014 atm**	-19.0 for 'y' -23.9 for 'x'	0.865 atm	**164**	1 for 'y' ½for 'x'
variation of repeated measurements	0.000132			153	
calibration standard	0.000050			58	

The bias, variance, and covariance of the least squares estimators was simulated for this situation where there are errors in both random variables. The procedure is as follows (Reilman et al., 1986) for i=1,...n (n=# of observations in the regression =7 for a 7-point calibration:

1) Generate random ui and vi according to a multivariate normal distribution with zero mean and covariance matrix V where the diagonal elements of V are the uncertainty for x and the uncertainty for y. There is covariance between the uncertainties due to the use of the same instrument for Pin and Pamb and since both x and y contain Pamb and Tamb. The correlation between the two and is conservatively assumed to be near 1.

2) Generate a random value for the nominal value of xi, given by Xi. This value is from a normal distribution with mean associated with the expected nominal values and variance from the expected variation in the ambient conditions.

3) Assume a linear relationship between the nominal values of Xi and Yi , given by Yi=a0+a1*Xi.

4) Calculate the observed values xi=Xi+ui and yi=Yi+vi .

5) Generate N data sets of size n and calculate mj (least squares slope estimate for data set j) and. bj (least squares intercept for data set j) where j=1,...,N.

6) Calculate the following for the least squares estimator in the above model:
   Slope bias = sb = (mean(mj)-a1)   Intercept Bias = ib = (mean(bj)-a0)
   Slope variance = sv = variance(mj)   Intercept variance = iv = variance(bj)
   Slope MSE = (sb*sb+sv)   Intercept MSE = (ib*ib+iv);
   Covariance = corr(mj,bj)*sqrt(sv)*sqrt(iv)

7) The MSE for the slope and the MSE for the intercept are used to represent the uncertainties in the slope (m) and intercept (b). The covariance estimate of the slope and intercept is accounted for in the overall uncertainty analysis.

Result:
The above procedure was performed at the nominal conditions found in the laboratory for a measured flow rate of 16.5 liters/minute. The total estimated uncertainty is ± 0.11 liters/minute which corresponds to ± 0.67%.

V. UNCERTAINTY ANALYSIS of FLOW RATE for a 16.7 liters/minute AIR SAMPLER

Following the methods employed the previous section, one can estimate the uncertainty in flow rate measured by air samplers in one's own air monitoring network using the standard uncertainties of the network's field transfer standards. The following example is done with standard uncertainties and sensitivity coefficients presented in normalized form (ppm) to facilitate presentation and calculations. If one's sampler calibration procedures incorporate a least squares fit of the calibration data, a linear structural model should be used as was presented in the previous section.

Note, as discussed earlier the Streamline uncertainty analysis assumed the discharge coefficient (Cd) and expansion factor (Y) to be constant in the laboratory. In practice Streamlines are used at a variety of temperature and barometric pressures and some uncertainty is therefore associated with these parameters. Conservative uncertainties for Cd and Y are thus included in this uncertainty analysis for a 16.7 liter/minute sampler.

The individual uncertainties estimated for Cd and Y are obtained from the tabulated empirical data[6,8] by estimating the variation of each of these parameters to changes in flow and ambient conditions (as Reynolds number). Variations were taken over extremes of ambient conditions and flow rates and can be considered worst case. The probability distribution of the tabulated data are assumed to be rectangular. Following are the assumed uncertainties in Cd and Y:

Discharge coefficient (Cd) = 0.29(13000) = 3770 ppm
Expansion factor (Y) = 0.29 (15000) = 4350 ppm

The uncertainty for a typical 16.7 liter/minute air sampler is summarized in Table 2. Individual standard uncertainties were estimated following the analysis in the previous section. These standard uncertainties were weighted with their normalized sensitivity coefficient, and the overall uncertainty obtained by quadrature sum.

Table 2: Summary of uncertainty components for the flow rate of a PM2.5 sampler.

Source of Uncertainty	Standard Uncertainty *u* [ppm]	Sensitivity Coefficient C	Product (C)(u) [ppm]
Streamline performance characteristics	**6700**	1	6700
Pressure drop across Streamline (DP)	**8393**	½	4197
variance of repeated observations (0.05"H2O)	8333		
calibration standard	1000		
Ambient temperature (Tamb)	**533**	½	267
variance of repeated observations (0.1K)	340		
calibration standard	410		
Ambient pressure (Pamb)	**529**	½	265
variance of repeated observations (0.01"Hg)	334		
calibration standard	410		
Discharge Coefficient (Cd)	**3770**	1	3770
Expansion Factor (Y)	**4350**	1	4350
Combined Uncertainty			**9787 = 0.98%**

Readers are encouraged to substitute estimates for the individual uncertainties of their particular field instrument to estimate the overall uncertainty of the samplers in their network. Such an analysis also allows a sensitivity evaluation of each instrument's contribution to overall flow rate measurement uncertainties. For example, in the above example the manometer used for measuring pressure drop across the Streamline contributes by far the most uncertainty of the electronic field instruments.

VI. CONCLUSIONS

An uncertainty analysis was performed for flow rates measured by the Streamline Flow Transfer Standard. Conservative decisions and estimates were use in all cases. The results indicate that Streamline FTSs are capable of providing flow rates with uncertainties of approximately ± 0.67%, or ± 0.11 liter/minute at 16.5 liters/minute.

Utilizing this result a similar analysis was performed for a typical 16.7 liter/minute air sampler. The analysis indicates that these samplers are capable of measuring flow rates with accuracies approximately ± 0.98%, or ± 0.16 liters/minute at nominal flow of 16.7 liters/minute.

Inspection of the relative contributions of component measurement uncertainties allows end users to select suites of instruments which combined will satisfy their quality assurance criteria.

VII. IMPLICATIONS

Flow rate is an integral component in the calculation of sample volumes for air pollution samplers in that sample volumes are a fundamental component in the determination of ambient pollutant concentrations. This paper provides insights and tools for estimating the accuracy of sample volumes collected by air samplers, and therefore for the accuracy of air pollution concentration measurements. This work is intended to assist regulators, air quality network managers and quality assurance personnel in determining the accuracy and effectiveness of ambient air quality monitoring programs.

VIII. KEY WORDS

Uncertainty analysis, Streamline Flow Transfer Standard, fine particle measurement, PM2.5, flow rate, flow metrology, air flow.

IX. REFERENCES

1. ANSI/NCSL Standard: ANSI/NCSL Z540-2-1997, "U.S. Guide to the Expression of Uncertainty in Measurement, published by the National Conference of Standards Laboratories, Boulder, CO, 1998. *Note; this is the U.S. version of the International Organization for Standardizations (ISO) "Guide to the Expression of Uncertainty,"1995.*
2. Kegel, Thomas M., "Uncertainty Analysis of a Sonic Nozzle Based Flowmeter Calibration, presented at the National Conference of Standards Laboratories, 1994 Workshop and Symposium, July 31- August 4, 1994.
3. ASME/ANSI Standard: MFC-7M-1987, "Measurement of Gas Flow by Means of Critical Flow Venturi Nozzles, an American National Standard published by the American Society of Mechanical Engineers, New York, N.Y., 1992.
4. Mattingly, G.E., "Fluid Flowrate Metrology", ISA Practical Guide for Flow Measurement, edited by D.W. Spitzer, Published by the Instrument Society of America, Research Triangle Park, NC, 1991.
5. Mattingly, G.E., "Gas Flowrate Metrology", National Conference of Standards Laboratories, **v29**, pp9-16, 1989.
6. ANSI/ASME Standard: MFC-2M-1983, "Measurement Uncertainty for Fluid Flow in Closed Conduits", an American National Standard published by the American Society of Mechanical Engineers, New York, N.Y., 1984.
7. Reilman, M. A. et. Al., "Stochastic Regression With Errors in Both Variables, Journal of Quality Technology, **v18**, No. 3, 1986.
8. "Flow of Fluids Through Valves, Fittings and Pipe, Technical Paper No. 410, Crane Company, New York, N.Y., 1982.
9. Belanger, B.C., "Traceability – an Evolving Concept", ASTM Standardization, Feb. 1979.
10. Arnberg, B.T., W. F. Siedl, "Discharge Coefficient Correlations for Circular-Arc Venturi Flowmeters at Critical (Sonic) Flow, Journal of Fluids Engineering, 1974.
11. Arnberg, B.T., "Practice and Procedures of Error Calculations", Symposium on Flow, Paper No. 3-7-215, Pittsburgh, PA, May10-14, 1971.
12. Arnberg, B.T., and C.L. Britton "Two Primary Methods for Proving Gas Flow Meters", Symposium on Flow, Paper No. 3-8-216, Pittsburgh, PA, May10-14, 1971.